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by

Lai Wu-yan, Wang Ding-sheng
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Dipole-Exchange Spin Waves in a Cylindrical Ferromagnet

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Abstract

Taking both dipole and exchange interaction into account, we have deduced the secular equation for the dipole-exchange spin waves in an axially magnetized cylindrical ferromagnet. Numerical results of frequency spectra are also given. When the wave vector the spin wave is small, dipole energy is the most important term and our results coincide with that of magnetostatic modes. When the wave vector becomes large, exchange energy plays a great role, and our results approach those of the theory of marcroscopic exchange waves.

1. Introduction

This article is intended to discuss the behavior of spin waves in an axially magnetized cylindrical ferromagnet when the dipole and exchange energy is of approximate magnitude. We call this kind of waves as dipole-exchange spin waves. For instance, in yttrium iron garbet (YIG), when wave vector $B \approx 10^5 cm^{-1}$, these two sets of energy are almost the same.

In a region where $\beta < 10^4 cm^{-1}$, diploe energy is the major one; it is the so-called magnetostatic mode region. Joseph and Schlomann have defined the magnetostatic mode in cylindrical ferromagnet. In a region where $\beta > 10^6 cm^{-1}$, exchange energy is the major one; this is the so-called exchange mode region. Herring and Kittel have found the spin wave

frequency spectra by using semi-classical method in their study of the theory of macroscopic spin waves. It can be possible to discuss frequency spectra within the range of a whole wave length only if both dipole and exchange interaction is taken into account at the same time. The solution of energy spectrum of dipole-exchange spin waves of plane film ferromagnet can be found only in de Wames and Wolfram's works.

Through discussion of dipole-exchange spin waves in a cyclindrical ferromagnet in this article, we can find frequency spectra of whole spin wave. When spin wave vector β is smaller, it is correspondent to magnetostatic mode. When β is larger, it approaches the result of exchange mode theory of macroscopic spin waves. In the middle region, spin wave is a mixture of magnetostatic mode and exchange mode, and at the same time it has body wave and surface wave. State density has noticeable changes, it cannot simply add magnetostatic energy and exchange energy up to substitute for whole frequency spectra.

2. Basic Methods

Here is a dipole-exchange spin wave model in an axially magnetized cylinder. When the wave length of spin waves is much longer than atomic separation, the spin waves can be described by motion equation of macroscopic magnetic moment. And the motion of macroscopic magnetic moment can satisfy magnetostatic equation (omit dissemination effect)

$$\mathrm{rot}\,\boldsymbol{h} = 0, \tag{2.1}$$

$$\mu_0 \mathrm{div}\,\boldsymbol{h} + \mathrm{div}\,\boldsymbol{m} = 0, \tag{2.2}$$

and motion equation of magnetic moment

$$\frac{d}{dt}\boldsymbol{M} = -\gamma \boldsymbol{M} \times \boldsymbol{H}, \tag{2.3}$$

In it, effective field $H = H_0 + h + \frac{1}{\mu_0} D\nabla^2 M$ is the sum of constant megnetic field H_0 (along cylindrical axis z), microscopic magnetic filed h and equavalent exchange field $\frac{1}{\mu_0} D\nabla^2 M$ (D is the parameter of exchange action, for example, in YIG, D = 2.6 x $10^{-12}cm^2$).

The relation between magnetic moment and time is $e^{j\omega t}$, taking time factor off, equation (2.3) is then written in component form as

$$\omega_0(1 - \alpha\Delta)m_x - j\omega m_y = \mu_0\omega_m h_x, \quad (2.4)$$

$$j\omega m_x + \omega_0(1 - \alpha\Delta)m_y = \mu_0\omega_m h_y, \quad (2.5)$$

in it $\omega_0 = \gamma H$, $\omega_m = \gamma M_0/\mu_0$, $\alpha = D\omega_m/\omega_0$.

Adding scalar quantity magnetic potential ψ, and making $h = -\nabla\psi$, then from equations (2.1)-(2.3), under linear approximation, a sextic differential equation of the internal magnetic potential ψ^{in} of the sample can be produced

$$\left[(\theta^2 - \Omega^2 + \theta)\nabla^2 - \theta \frac{\partial^2}{\partial z^2}\right]\psi^{in} = 0, \quad (2.6)$$

in it θ is a differential operator, $\theta = \Omega_m^{-1}(1 - \alpha\nabla^2)$, and constant $\Omega = \omega/\omega_m$, $\Omega_m = \omega_m/\omega_0$.

On the outside of the sample, magnetic potential ψ^{out} satisfies equation

$$\nabla^2\psi^{out} = 0. \quad (2.7)$$

For a axially magnetized cylinder, the solution of equation (2.6) should be of the following form:

$$\psi^{in} = \int_{-\infty}^{\infty} d\beta e^{j\beta z}\Phi_\beta(\rho, \varphi). \quad (2.8)$$

Substituting equation (2.8) for equation (2.6), we have $\Phi_\beta(\rho, \varphi)$, by which the equation can be satisfied is

$$(\Delta_t + k_1^2)(\Delta_t + k_2^2)(\Delta_t + k_3^2)\Phi_\beta(\rho, \varphi) = 0, \tag{2.9}$$

in it $\Delta_t = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\rho^2}$, k_1^2, k_2^2, k_3^2 are three roots of a cubic algebra equation of k

$$(k + \beta^2)^3 + \frac{2 + \Omega_m}{\alpha}(k + \beta^2)^2 + \frac{1 - \Omega^2\Omega_m^2 + \Omega_m - \alpha\beta^2\Omega_m}{\alpha^2}(k + \beta^2) - \frac{\beta^2\Omega_m}{\alpha^2} = 0 \tag{2.10}$$

It is easy to see that the solution of equation (2.9) is the sum of solutions of three Helmholtz equation s $(\Delta_t + k_i^2)\Phi = 0, (i = 1,2,3)$. And thereupon the general solution of equation (2.6) is

$$\psi^{in} = \sum_n \int_{-\infty}^{\infty} d\beta \sum_{i=1}^{3} C_{n\beta}^i J_n(k_i\rho) e^{i(\beta z - n\varphi)} \quad \rho < a, \tag{2.11}$$

in it a is radius of the cylinder and $J_n(k_i\rho)$ is n 阶 $(n = 0, \pm 1, \pm 2 \cdots)$ Bessel function. The items indicated by Neumann function N_n(~~kip~~ $k_i\rho$) in general solution, because of the limited boundary condition $\psi^{in}(\rho = 0)$, are exluded.

For the outside of the sample, the general solution of equation (2.7) is

$$\psi^{out} = \sum_n \int_{-\infty}^{\infty} d\beta D_{n\beta} K_n(\beta\rho) e^{i(\beta z - n\varphi)}, \tag{2.12}$$

in it $K_n(\beta\rho)$ is transformed Bessel function of the second kind. $I_n(\beta\rho)$ indicated by transformed Bessel function of first kind in general solution, because boundary condition $\psi^{out}(\rho \to \infty) = 0$, are excluded.

3. Boundary Condition

The corresponding boundary conditions at $\rho = a$ of magnetostatic equations (2.1) and (2.2) are tangential component succession of h and

normal component succession of $\mu_0 h + m$, namely

$$\phi^{in}|_{\rho=a} = \phi^{out}|_{\rho=a}, \tag{3.1}$$

$$(\mu_0 h_\rho^{in} + m_\rho)|_{\rho=a} = \mu_0 h_\rho^{out}|_{\rho=a}. \tag{3.2}$$

For magnetostatic mode, $\alpha = 0$, equations (2.4) and (2.5) are transformed into algebra equations, and only equations (2.1) and (2.2) remain to be differential equations, and there are only two coefficients in their corresponding solutions. The magnetostatic secular model can be determined by these two boundary conditions.

For dipole-exchange mode, there must be boundary conditions corresponding to differential equations (2.4) and (2.5). There is a frequently used boundary condition which assumes that the boundary magnetic moment is same as the internal magnetic moment, and can satisfy same motion equation. This means that the surface magnetic moment is completely "free". Thus from equations (2.4) and (2.5) and by using generalized function method, if boundary plane is proved to be a cylindrical plane, the "free" boundary condition is

$$\left(\frac{\partial}{\partial\rho} + \frac{1}{\rho}\right) m|_{\rho=a} = 0. \tag{3.3}$$

Another frequently used boundary condition makes $m(\rho = a) = 0$. This means that, because of some reason, it makes magnetic moment of the boundary plane completely "nailing upon" z direction. It may use a certain combination (partial nailing up) of the two conditions. In Document (3), in the discussion of dipole-exchange spin waves of plane film, the surface is considered as a crystal plane. Then through the process of microscopic motion equation approaching to succession, a corresponding boundary condition is obtained.

The merit of such a solution is that when it is short wave, it is possible to approach an exchange mode that is acquired through the solution of a microscopic motion equation. But it is difficult to extend this method to cylindrical surface. The operation that follows is to use "free" boundary condition, and the impact of boundary conditions will be discussed in Section 5.

4. Secular Equation

The solution ψ of equations (2.11) and (2.12) can satisfy equation (2.6). From $\boldsymbol{h} = -\nabla\psi$, it is possible to secure a corresponding h. To seek m from h, it should be under boundary equation (3.3) to try to solve differential equations (2.4) and (2.5). Using Green function method, it can have

$$\begin{pmatrix} m_\rho \\ m_\varphi \end{pmatrix} = \frac{\mu_0 \Omega_m}{2\alpha} \sum_n \int_{-\infty}^{\infty} d\beta \sum_{i=1}^{3} C^i_{n\beta} e^{i(\beta z - n\varphi)} \times \begin{pmatrix} 1 & -1 \\ -j & -j \end{pmatrix} \begin{pmatrix} \frac{k_i}{k_i^2 - \xi^2}[J_{n+1}(k_i\rho) - B_i^+ J_{n+1}(\xi\rho)] \\ \frac{k_i}{k_i^2 - \eta^2}[J_{n-1}(k_i\rho) - B_i^- J_{n-1}(\eta\rho)] \end{pmatrix}, \tag{4.1}$$

in it

$$\xi^2 = \frac{1}{\alpha}(\Omega\Omega_m - 1) - \beta^2, \quad \eta^2 = -\frac{1}{\alpha}(\Omega\Omega_m + 1) - \beta^2,$$

$$B_i^+ = \frac{k_i a J'_{n+1}(k_i a) + J_{n+1}(k_i a)}{\xi a J'_{n+1}(\xi a) + J_{n+1}(\xi a)}, \quad \cdot B_i^- = \frac{k_i a J'_{n-1}(k_i a) + J_{n-1}(k_i a)}{\eta a J'_{n-1}(\eta a) + J_{n-1}(\eta a)}.$$

In order to make solution of equation (4.1) satisfy equation (2.2), it can prove that coefficient $C^i_{n\beta}$ should be made satisfy

$$\sum_{i=1}^{3} C^i_{n\beta} \frac{k_i}{k_i^2 - \xi^2} B_i^+ = 0. \tag{4.2}$$

$$\sum_{i=1}^{3} C^i_{n\beta} \frac{k_i}{k_i^2 - \eta^2} B_i^- = 0. \tag{4.3}$$

then

$$\begin{pmatrix} m_\rho \\ m_\varphi \end{pmatrix} = \frac{\mu_0 \Omega_m}{2\alpha} \sum_n \int_{-\infty}^{\infty} d\beta \sum_{i=1}^{3} C^i_{n\beta} e^{i(\beta z - n\varphi)}$$

$$\times \begin{pmatrix} 1 & -1 \\ -j & -j \end{pmatrix} \begin{pmatrix} \dfrac{k_i J_{n+1}(k_i \rho)}{k_i^2 - \xi^2} \\ \dfrac{k_i J_{n-1}(k_i \rho)}{k_i^2 - \eta^2} \end{pmatrix}. \tag{4.4}$$

By substituting equations (2.11), (2.12) and (4.4) for boundary condition equation (3.1) and (3.2), the coefficient of ψ acquired should satisfy the other two conditions which are

$$\sum_{i=1}^{3} C^i_{n\beta} J_n(k_i a) = D_{n\beta} K_n(\beta a), \tag{4.5}$$

$$\sum_{i=1}^{3} C^i_{n\beta}\left(k_i a J'_n(k_i a) - \frac{\Omega_m}{2\alpha} k_i a \left[\frac{J_{n+1}(k_i a)}{k_i^2 - \xi^2} - \frac{J_{n-1}(k_i a)}{k_i^2 - \eta^2}\right]\right) = D_{n\beta} \cdot \beta a K'_n(\beta a). \tag{4.6}$$

The non-zero solution condition of the linear equations (4.2), (4.3), (4.5) and (4.6) of coefficient of ψ is the secular equation that determines dipole-exchange mode.

From equations (4.2) and (4.3) it can have

$$C^2_{n\beta} = \frac{E(3,1)}{E(2,3)} \cdot \frac{J_n(k_1 a)}{J_n(k_2 a)} \cdot C^1_{n\beta},$$
$$C^3_{n\beta} = \frac{E(1,2)}{E(2,3)} \cdot \frac{J_n(k_1 a)}{J_n(k_3 a)} \cdot C^1_{n\beta}. \tag{4.7}$$

Substituting the above equation for (4.5) and (4.6), it can have linear equation of $C^i_{n\beta}$ & $D_{n\beta}$. The non-zero solution condition is

$$\frac{L(1)E(2,3) + L(2)E(3,1) + L(3)E(1,2)}{E(1,2) + E(2,3) + E(3,1)} = \frac{\beta a K'_n(\beta a)}{K_n(\beta a)}, \tag{4.8}$$

in it

$$L(i) = \mu_i \frac{k_i a J'_n(k_i a)}{J_n(k_i a)} - n\chi_i,$$

$$E(i,j) = \begin{vmatrix} F_i & F_j \\ G_i & G_j \end{vmatrix},$$

$$F_i = n(\mu_i - 1)\frac{k_i a J'_n(k_i a)}{J_n(k_i a)} + [(k_i a)^2 - n^2]\chi_i,$$

$$G_i = n\chi_i \frac{k_i a J'_n(k_i a)}{J_n(k_i a)} - [(k_i a)^2 - n^2](\mu_i - 1),$$

$$\mu_i = -\beta^2/k_i^2,$$

$$\chi_i = \Omega\Omega_m(1 - \mu_i)/[1 + \alpha(k_i^2 + \beta^2)] \qquad i, j = 1,2,3.$$

Under the given Ω_m, a, β and n, Ω which can satisfy equation (4.8) is the secular frequency of dipole-exchange mode.

5. Results and Discussions.

The three roots of equation (2.10) corresponding to potential equation (2.11) and magnetic moment distribution equation (4.4) have three oscillations, and the secular equation of dipole-exchange mode is the accumulation of these three oscillations. When $\Omega < \Omega_m^{-1}(1 + \alpha\beta^2)$, k_1^2, k_2^2 & k_3^2 are all negative . But within this range, to secular equation (4.8) there is no solution. When $\Omega \geqslant \Omega_m^{-1}(1 + \alpha\beta^2)$, $k_1^2 \geqslant 0$, k_2^2, $k_3^2 < 0$, and this indicates that of the three oscillations which are included in the secular equation, one is body mode and the other two are surface mode.

Under the limit of long wave, when $\beta \ll 1/\sqrt{\alpha}$, and to the range of $\Omega_m^{-1}(1 + \alpha\beta^2) < \Omega < \Omega_m^{-1}(1 + \Omega_m)^{1/2}$, $k_1^2 \approx \beta^2(\Omega^2 - \Omega_m^{-2})/(\Omega_m^{-2} + \Omega_m^{-1} - \Omega^2)$. This means that k_1 and the magnetostatic body mode are the same. To the range of $\Omega > \Omega_m^{-1}(1 + \Omega_m)^{1/2}$, $k_2^2 \approx \beta^2(\Omega^2 - \Omega_m^{-2})/(\Omega_m^{-2} + \Omega_m^{-1} - \Omega^2)$. This indicates that k_2 approaches magnetostatic surface mode.

k_3 represents one surface mode, and it will quickly decay following the increase of its distance from the surface. Because $k_3^2 < k_3^2(\Omega = 0) = -(\beta^2 + 1/\alpha + \Omega_m^{-1}/\alpha)$, it concentrates only in the surface layer, of which the thickness is about $\sqrt{\alpha}$ level of quantity. Among the magnetostatic modes, there is no one which can be same as this one. $\sqrt{\alpha}$ has a length net, which marks the characteristic length of exchange action. To YIG, when $\Omega_m = 2$, $\sqrt{\alpha} = \sqrt{\Omega_m D} = 2.3 \times 10^{-6}$cm.

Figure 1 is the relationship of k_1^2, k_2^2, k_3^2 & Ω, which are secured from the solution of equation (2.10), when $\Omega_m = 2,\ \beta = 0.1(1/\sqrt{\alpha})$. Figure 2 is a contrast between $(-\mu_1)^{-1}$ & $(-\mu_2)^{-1}$ and $(-\mu)^{-1} = (\Omega^2 - \Omega_m^{-2})/(\Omega_m^{-2} + \Omega_m^{-1} - \Omega^2)$ of magnetostatic mode (the symbols used in Figure 1 and Figure 2 are K_1^2 & K_1^2/β^2). They clearly represent the results of the foregoing discussions.

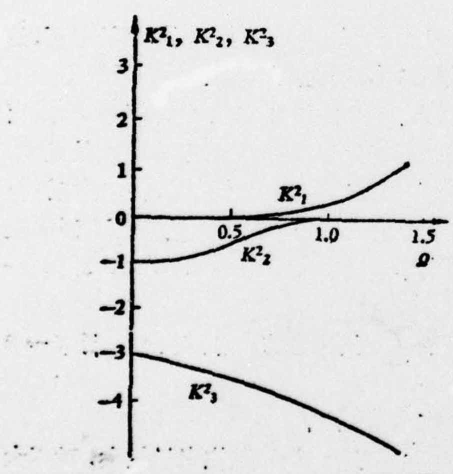

Figure 1 The three oscillations of dipole-exchange mode
$\Omega_m = 2,\ \beta = 0.1(1/\sqrt{\alpha})$

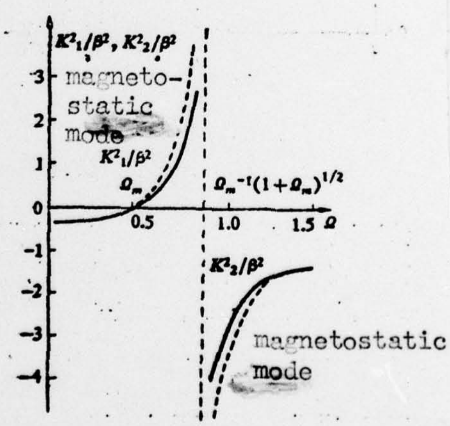

Figure 2 Comparison of dipole-exhange mode and magnetostatic mode
$\Omega_m = 2,\ \beta = 0.1(1/\sqrt{\alpha})$

Under given β and n, the solution of secular equation (4.8) gives a series of eigenvalue. $\Omega_{\beta nr}$, $r = 1, 2, 3\cdots$ is the order number of eigenvalue, and different r represents different zero number which occurs when potential function ψ at the region where $\rho = [0, a]$.

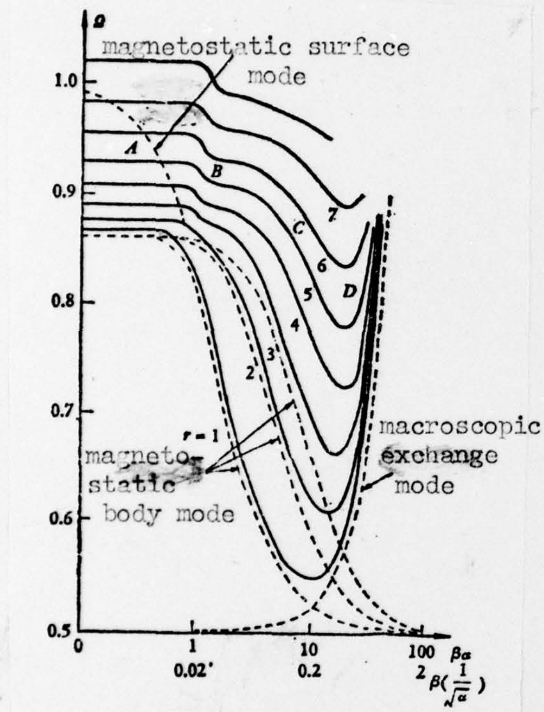

Figure 3 Dipole-exchange mode frequency spectra (solid line)
$\Omega_m = 2$, $a = 50\sqrt{\alpha}$, $n = 1$

Figure 3 is the eigenvalue $\Omega_{\beta nr}(r = 1, 2, \cdots 7)$, of dipole-exchange mode obtained from the solution of equation (4.8) when $\Omega_m = 2$, $a = 50\sqrt{\alpha}$, $n = 1$. At the same time, it gives spin wave (exchange mode) frequency spectra of macroscopic spin theory. For the situation of wave vector paralleling with magnetic direction (exchange) Ω_β (交换) $= \Omega_m^{-1}(1 + \alpha\beta^2)$, for comparison, it gives three curves ($r = 1, 2, 3$) in magnetostatic modes of cylindrical ferromagnet.

At short wave, when $\beta \gg n(1/\sqrt{\alpha})$ and $\beta \gg r(1/\sqrt{\alpha})$, the theoretical results of dipole-exchange mode and macroscopic spin wave exchange mode are the same.

At long wave, when β is small and if the cylindrcal radius is very large, the contribution of exchange energy to n and r is very small, and the

energy spectra and magnetostatic modes are the same. When the cylindrical radius is not very large (as $a = 50\sqrt{\alpha}$ in Figure 3), the impact of exchange energy can already be seen. And the dipole-exchange modes are probably already in shape of

$$\Omega_{\beta nr} = \Omega_{\beta nr} \text{ (magnetostatic) + exchange correction} \quad (5.1)$$

In a region where $1/a \ll \beta \ll 1/\sqrt{\alpha}$, k_2^2 acquired from the solution of equation (2.10) includes $|k_2^2| > \beta^2 \gg 1/a^2$. So oscillation k_2 can only have some action on the surface of the sample, and its contribution to exchange energy is not great. The exchange correction in equation (5.1) comes mainly from k_1, so exchange correction $\approx \alpha \Omega_m^{-1} k_1^2$.

Roughly speaking, the distribution of eigenvalue probably is $k_1 a \approx \left(r + \frac{n}{2}\right)\pi$, so, only if $a \gg \sqrt{\alpha}$ and under the situation that n and r are not vrey large, the correction is but a small one. This is just the situation indicated in Figure 3.

The conventional methods are simply to take spin wave spectra as an accumulation of magnetostatic energy spectra and exchange energy spectra, and give an analogue result to equation (5.1). From strict computation, it is known that it can be established only when β is larger (short wave) or when β is smaller (long wave). Then there is no great difference from the rigid theories. But this cannot be applied to the medium range, $\beta \sim 10^4$—10^6/cm, namely dipole-exchange mode range.

When $\beta \ll 1/a$, the difference between dipole-exchaneg mode and magnetostatic mode is very clear. In magnetostatic mode, when $\beta \to 0$, all

frequency spectra of n and r will be

$$\Omega_{\beta nr}(\text{静磁})\big|_{\beta=0} = \Omega_m^{-1}(1+\Omega_m)^{1/2}.$$

After considering exchange action, there will be no more combination of different n and r. If the cylindrical radius is very large, to the small n and r, this split is not noticeable. But to radius a, which is not large, as $a = 50\sqrt{\alpha}$ in Figure 3, this phenomenon cannot be overlooked. Besides, in the range of $\beta \ll 1/a$, when Ω is large and $|k_2^2|$ has been reduced to below $1/a^2$, so the exchange energy of surface mode k_2 becomes important. After considering exchange action, secular function is always a mixture of body mode and surface mode and this kind of mixture will become especially remarkable when intersecting of body mode and surface mode takes place. In Ω-β, surface mode is no longer a single curve, and it has been cut into several sections which connect the body mode (Figure 3). Taking dipole-exchange mode r = 5 as example, in section A, body mode (k_1) is the major part mixed with surface mode; in section B, surface mode (k_2) is the major part mixed with body mode; in section C, body mode is the major part, and following the increase of β, the surface mode is continuously reduced; in section D, when it approached short wave, exchange energy, after surpassing magnetostatic energy, gradually becomes into exchange body mode.

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